

Hunting for the Quantum Cheshire Cat

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Abstract

The proposal of Aharonov, Popescu, and Skrzypczyk [arXiv:1202.0631] of disembodiment of physical properties from particles is analyzed. It is argued that: (1) in order to state that the cat is at one location and the smile at another, one should look at correlations, not mean values; (2) a weak value of one for the presence of the cat describes the average over a large number of trials, where the detector gives in each trial outputs that are not zero nor one, and that are much larger than unity (they can be large and negative as well); (3) once these issues are addressed, the specific model proposed does not provide evidence for disembodiment of physical properties. Here, the exact probability distribution and its characteristic function are derived for arbitrary coupling strength, preparation and post-selection. This allows to successfully hunt down the quantum Cheshire cat.

1 Introduction

Weak measurement followed by postselection [1] is a powerful inference technique, allowing e.g. to reconstruct the unknown wavefunction of a system [2], or the density matrix [3–5]. Perhaps “measurement” is a misleading term, since due to the weak interaction between system and meter one cannot infer substantial information from a single trial. However, the statistical analysis of the postselected data — which generally is limited to the average readout but could be extended to the full statistics [6] — allows to extract information about the system that is not trivially recovered from standard strong projective measurements. Furthermore, since the quantum coherent nature of the meter is of the essence [7], it is possible to focus on the meter itself and use the system as an ancilla, e.g., to provide signal amplification [8,9]. For recent reviews of the weak measurement, see Refs. [10–12].

Recently, Ref. [13] proposed a way to realize a “Cheshire cat” by using joint weak measurements of commuting observables of a photon. The claim

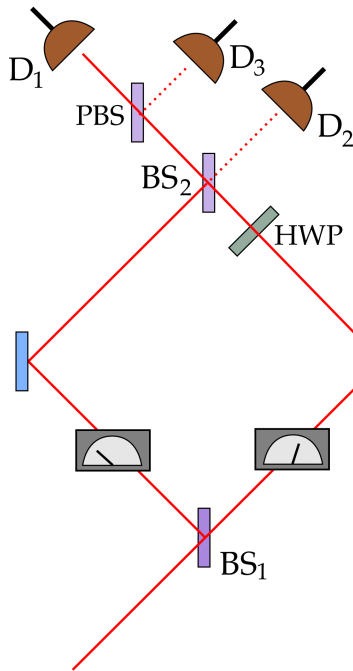


Figure 1: The setup considered (reproduced from Ref. [13]).

is that the smile of the cat (the polarization of the photon) is at one arm of the interferometer (where the photon is measured) while the cat (the photon itself) is at the other arm. Two related papers quickly followed up [14, 15], addressing further issues. Here we show that for the specific model proposed in Ref. [13], the disembodiment of physical properties from the physical vehicle cannot occur. By extending the results to any preparation and post-selection, and to any coupling strength, we succeed in individuating a region where the Cheshire cat can actually be found and we proceed to its capture. To this goal, we study the exact statistics of the measurement readouts, and consider the covariance between them for any interaction strength.

2 Review of the proposed setup

With reference to Fig. 1, from bottom to top, a horizontally polarized photon impinges on a non-polarizing beam splitter (BS1), and is thus prepared in the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|L, H\rangle + |R, H\rangle), \quad (1)$$

with L, R referring to the spatial part of the photon propagating, respectively, in the left and right arm. In the left arm a meter measures either the presence

of the photon, characterized by the observable

$$\Pi_L = 1|L\rangle\langle L| + 0|R\rangle\langle R|, \quad (2)$$

or its circular polarization

$$\sigma_L = |L, +\rangle\langle L, +| - |L, -\rangle\langle L, -|. \quad (3)$$

(We recall that the relation between the normalized eigenstates of horizontal and vertical polarization and the eigenstates of circular polarization is $|H\rangle = (1/\sqrt{2})(|+\rangle + |-\rangle)$ and $|V\rangle = (1/\sqrt{2})(|+\rangle - |-\rangle)$.) Analogously a meter in the right arm measures either Π_R or σ_R . Then, the two paths are recombined, after the right one passes through a half-wave plate (HWP) that takes a state $|V\rangle$ to $|H\rangle$, into a second non-polarizing beam splitter (BS2), that is engineered to take the state $|L\rangle + |R\rangle$ to the left $|L'\rangle$, and the orthogonal state $|L\rangle - |R\rangle$ to the right $|R'\rangle$ (in which case a photon counter D2 will click). Finally, a polarizing beam splitter will take the state $|L', H\rangle$ to the left $|L''\rangle$, and $|L', V\rangle$ to the right. This way, if the photon detector D1 clicks, we can be sure that the state of the photon, just before crossing HWP, is

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|L, H\rangle + |R, V\rangle). \quad (4)$$

This is the post-selected state. By analyzing data corresponding to a click in the photon detector D_1 it is possible to access the conditional probability of observing readouts X, Y in the meters, given that the system was prepared in $|\Psi\rangle$ and post-selected in $|\Phi\rangle$.

3 Analysis of the proposal of Ref. [13]

We study the statistics of the readouts for the setup described in the previous section. We focus on the case when the polarization σ_R in the right arm and the presence Π_L in the left arm are measured. The interaction Hamiltonian is

$$H = \delta(t - t_L)a\hat{P}_X \sum_{\sigma} |L, \sigma\rangle\langle L, \sigma| + \delta(t - t_R)b\hat{P}_Y \sum_{\sigma} \sigma |R, \sigma\rangle\langle R, \sigma|, \quad (5)$$

with a, b having the dimension of a length and describing the shift in the beam, while \hat{P}_j are the components of the momentum. The times of interaction are determined by the passage of the Z variable (describing the forward propagation of the beam) at the location of the detectors. We assume that the photon is well localized along the direction of propagation, and that its wavefunction does not spread appreciably in this direction, so that $c\delta(Z - Z_L)$ can be effectively replaced by $\delta(t - t_L)$. Since the meter is assumed to give a continuous output, the weakness of the measurement can be realized by having the initial distribution of the readout variables (the conjugate variables of \hat{P}_X and \hat{P}_Y , i.e. the positions \hat{X} and \hat{Y}) to have a coherence¹ $\delta \gg \max[a, b]$. The meters are

¹The spread is, by the uncertainty principle, $\Delta \geq \delta/2$, see e.g. Ref. [16].

assumed to be initially uncorrelated between them (otherwise they could give false positives), and with the system (otherwise it would be problematic to say we have a measurement at all), so that the density matrix before the interaction is

$$\hat{R} = |\Psi\rangle\langle\Psi| \otimes \hat{\rho}_X \otimes \hat{\rho}_Y. \quad (6)$$

After applying Born's rule, the joint probability of making a successful postselection in $|\Phi\rangle$ and observing the outputs X, Y is

$$Prob\{\Phi, X, Y|\Psi, \rho_X, \rho_Y\} = \langle\Phi, X, Y|U\hat{R}U^\dagger|\Phi, X, Y\rangle, \quad (7)$$

with U the time evolution operator. Let $\Pi_X(X) \equiv \langle X|\hat{\rho}_X|X\rangle$ and $\Pi_Y(Y) \equiv \langle Y|\hat{\rho}_Y|Y\rangle$ the initial distribution of the readouts. It is required that the meters are unbiased² so that $\int dX X \Pi_X(X) = \int dY Y \Pi_Y(Y) = 0$. Simple algebra applied to Eq. (7) gives

$$\begin{aligned} Prob\{\Phi, X, Y|\Psi, \rho_X, \rho_Y\} = & \frac{1}{4} \left\{ \Pi_X(X-a) \Pi_Y(Y) \right. \\ & + \frac{1}{4} \Pi_X(X) [\Pi_Y(Y-b) + \Pi_Y(Y+b)] \\ & + \frac{1}{2} [\rho_X(X-a, X) \rho_Y(Y, Y-b) + c.c.] \\ & - \frac{1}{2} [\rho_X(X-a, X) \rho_Y(Y, Y+b) + c.c.] \\ & \left. - \frac{1}{4} \Pi_X(X) [\rho_Y(Y+b, Y-b) + c.c.] \right\}. \quad (8) \end{aligned}$$

The last three lines describe interference in the readout, and vanish for a strong measurement. The conditional probability is obtained by dividing Eq. (8) by the marginal probability of a successful postselection

$$Prob\{\Phi|\Psi, \rho_X, \rho_Y\} = \int dX dY Prob\{\Phi, X, Y|\Psi, \rho_X, \rho_Y\}. \quad (9)$$

The initial state of the meters is assumed for simplicity a pure Gaussian with zero average momentum

$$\rho_X(X, X') = \frac{1}{\sqrt{2\pi}\Delta_X} \exp[-(X^2 + X'^2)/4\Delta_X^2] \quad (10)$$

$$\rho_Y(Y, Y') = \frac{1}{\sqrt{2\pi}\Delta_Y} \exp[-(Y^2 + Y'^2)/4\Delta_Y^2]. \quad (11)$$

In this case, there is an exact analytic solution (see Ref. [6] for a general treatment). The probability of postselection is

$$Prob\{\Phi|\Psi, \rho_X, \rho_Y\} = \frac{1}{8} [3 - \exp(-\varepsilon^2/2)] \quad (12)$$

²A systematic error indeed, could induce to believe that there was a disembodiment, e.g., if the presence detector gives prevalently a positive reading even when there is no photon.

with $\gamma = a/\Delta_X$, $\varepsilon = b/\Delta_Y$, so that the conditional probability for the normalized output $x = X/a, y = Y/b$ is

$$\begin{aligned} Prob\{x, y|\Phi, \Psi, \rho_X, \rho_Y\} &= \frac{\gamma\varepsilon}{[3 - \exp(-\varepsilon^2/2)]\pi} \\ &\times \left\{ e^{-\gamma^2(x-1)^2/2 - \varepsilon^2 y^2/2} \right. \\ &\quad + \frac{1}{4} e^{-\gamma^2 x^2/2} \left[e^{-\varepsilon^2(y-1)^2/2} + e^{-\varepsilon^2(y+1)^2/2} \right] \\ &\quad + e^{-(\gamma^2 + \varepsilon^2)/8} e^{-\gamma^2(x-1/2)^2/2} \left[e^{-\varepsilon^2(y-1/2)^2/2} - e^{-\varepsilon^2(y+1/2)^2/2} \right] \\ &\quad \left. - \frac{1}{2} e^{-\varepsilon^2/2} e^{-\gamma^2 x^2/2 - \varepsilon^2 y^2/2} \right\}. \end{aligned} \quad (13)$$

The characteristic function [6] is

$$\begin{aligned} Z(\chi, \eta) &\equiv \int dx dy e^{i(\chi x + \eta y)} Prob\{x, y|\Phi, \Psi, \rho_X, \rho_Y\} \\ &= \frac{2}{3 - \exp(-\varepsilon^2/2)} Z_0(\chi, \eta) \\ &\quad \times \left[e^{i\chi} + \frac{1}{2} \cos(\eta) + 2ie^{-(\gamma^2 + \varepsilon^2)/8} \sin(\eta/2) e^{i\chi/2} - \frac{1}{2} e^{-\varepsilon^2/2} \right], \end{aligned} \quad (14)$$

with the initial characteristic function

$$Z_0(\chi, \eta) = e^{-\chi^2/2\gamma^2 - \eta^2/2\varepsilon^2}. \quad (15)$$

The appearance of half-counting fields is typical of interference phenomena [17–20]

3.1 Strong measurement limit

This limit is obtained for $\gamma \rightarrow \infty, \varepsilon \rightarrow \infty$ and it is, as expected, a discrete probability:

$$p_{x,y|\Phi,\Psi,\rho_X,\rho_Y}^\infty = \frac{2}{3} \left[\delta_{x,1} \delta_{y,0} + \frac{1}{4} \delta_{x,0} (\delta_{y,1} + \delta_{y,-1}) \right]. \quad (16)$$

The characteristic function is $Z^\infty(\chi, \eta) = \frac{2}{3} [e^{i\chi} + \frac{1}{2} \cos(\eta)]$. As the limit is a bit tricky, we sketch how to obtain it: For large but finite γ, ε the interference terms in Eq. (13) (last two lines) are negligible. The remaining leading terms are a combination of products of Gaussian distributions centered at $x = 0, 1$ and $y = 0, 1, -1$, with spreads $1/\gamma$ and $1/\eta$. The output is discretized by identifying the points in the intervals $[x_0 - \Delta x/2, x_0 + \Delta x/2]$, with an arbitrary $\Delta x = 1/N$, N a positive natural number. The relevant terms are of the form

$$p_{Xn} \approx \frac{\gamma}{\sqrt{2\pi}} \int_{n/N - \Delta x/2}^{n/N + \Delta x/2} dx e^{-\gamma^2(x-\mu)^2}, \quad (17)$$

with $\mu = 0, 1$ (and analogously for y , with $\mu = -1, 0, 1$), and they tend to zero in the limit $\gamma \rightarrow \infty$, unless n is such that $\mu \in [n/N - \Delta x/2, n/N + \Delta x/2]$, in which case they tend to one. By putting finally $\Delta x \rightarrow 0$, the discrete distribution is recovered.

3.2 Weak measurement limit

This limit is obtained for $\gamma \rightarrow 0, \varepsilon \rightarrow 0$. The probability is uniformly distributed in x, y , and the characteristic function is $Z^0(\chi, \eta) = \delta_{\chi,0} \delta_{\eta,0}$. Trivially, if the meters have an initial infinite spread, or if the coupling constant is set to zero, the readout gives no information. However, the weak measurement is obtained by having a weak interaction, collecting data for the statistics, and then extrapolating to the limit of vanishing interaction. This reflects in the fact that, as Eq. (14) is not analytic as a function of ε, γ , the following limits do not commute:

$$\lim_{\substack{\chi \rightarrow 0 \\ \eta \rightarrow 0}} \frac{\partial^{j+k}}{\partial^j \chi \partial^k \eta} \lim_{\substack{\gamma \rightarrow 0 \\ \varepsilon \rightarrow 0}} Z(\chi, \eta) \neq \lim_{\substack{\gamma \rightarrow 0 \\ \varepsilon \rightarrow 0}} \lim_{\substack{\chi \rightarrow 0 \\ \eta \rightarrow 0}} \frac{\partial^{j+k} Z(\chi, \eta)}{\partial^j \chi \partial^k \eta}. \quad (18)$$

The right hand side corresponds to the experimental procedure, and gives finite values for the moments of the distribution.

3.3 Arbitrary strength

It is convenient to introduce $w = \exp[-\gamma^2/2]$ and $z = \exp[-\varepsilon^2/2]$. The average values are

$$\langle x \rangle = \frac{2}{3-z}, \quad (19)$$

$$\langle y \rangle = \frac{2(wz)^{1/4}}{3-z}, \quad (20)$$

$$\langle xy \rangle = \frac{(wz)^{1/4}}{3-z}, \quad (21)$$

The variances are

$$\langle\langle x^2 \rangle\rangle = \frac{1}{\gamma^2} + 2 \frac{1-z}{(3-z)^2}, \quad (22)$$

$$\langle\langle y^2 \rangle\rangle = \frac{1}{\varepsilon^2} + \frac{3-z-4(wz)^{1/2}}{(3-z)^2}, \quad (23)$$

$$\langle\langle xy \rangle\rangle = -(wz)^{1/4} \frac{1+z}{(3-z)^2}. \quad (24)$$

Ref. [13], considered that in the weak measurement limit $\langle x \rangle = 1$ and $\langle y \rangle = 1$. From this it was inferred that the photon is in the left arm, while its polarization is in the right arm. However, this inference does not take into account that in each individual repetition of a weak measurement, since the initial spread of the meter is much larger than the readout scale, the observed normalized readout

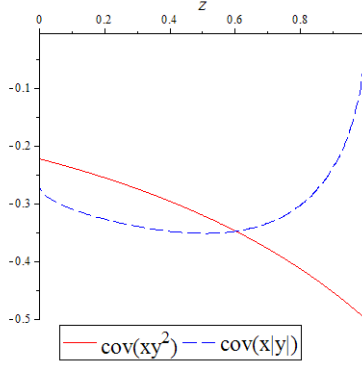


Figure 2: The covariances as a function of $z = \exp(-\varepsilon^2/2)$. In the weak limit both stay finite, even though the variances diverge, and they are negative for any coupling strength. For a strong measurement $z = 0$ the two covariances coincide, however, due to the parametrization chosen, $\langle x|y| \rangle$ raises very steeply to $-2/9$, so that one cannot see this from the figure.

is much larger than one (in absolute value). Even if the average over the post-selected data yields one, this does not warrant the conclusion that all photons passed in the left arm or all were polarized $+$ in the right arm. The only way one can ascertain whether the polarization is disembodied from the presence of the photon is to study the covariance of the two observables. Since the polarization takes values ± 1 (in addition to the initial value of the meter, which is zero just on the average, not in each individual trial), it is more sensible to consider the covariance between x and y^2 . In the strong measurement limit, the covariance is negative, since y^2 is one when $x = 0$ and *vice versa*. Precisely $\text{cov}(xy^2) = -2/9$. If there was an actual disembodiment of physical properties, this covariance should be positive. However, from Eq. (13) or Eq. (14) it follows that

$$\text{cov}(xy^2) = -\frac{2}{(3-z)^2}. \quad (25)$$

Thus the covariance stays negative for a weak measurement ($z \rightarrow 1$), as shown in Fig. 2.

Finally, let us consider the covariance between x and $|y|$. In the strong limit $|y| = y^2$, since the only possible outputs are $-1, 0, 1$. However for a measurement of intermediate strength, this is a distinct quantity from $\text{cov}(xy^2)$. Here, the characteristic function is of no help, and one should use directly Eq. (13), which gives

$$\langle |y| \rangle = \frac{1}{3-z} \left[\frac{2}{\sqrt{-\pi \ln z}} + \text{erf} \left(\sqrt{-\ln z} \right) \right], \quad (26)$$

$$\langle x|y| \rangle - \langle x \rangle \langle |y| \rangle = \frac{2}{(3-z)^2} \left[\frac{1-z}{\sqrt{-\ln z}} - \text{erf} \left(\sqrt{-\ln z} \right) \right] \quad (27)$$

with $\text{erf}(\cdot)$ the error function. The covariance given in Eq. (27) tends to zero for a weak measurement, but stays always negative, see Fig. 2. Therefore, it is concluded that with the preparation and post-selection considered in Ref. [13], there is no evidence for the presence of a quantum Cheshire cat. In the next section, we extend the analysis to arbitrary initial and final states.

4 Arbitrary preparation and post-selection

Let the hunt for the quantum Cheshire cat begin. The general prepared state is

$$|\Psi\rangle = \sum_{j,\sigma} A_{j,\sigma} |j, \sigma\rangle \quad (28)$$

and the post-selected state

$$|\Phi\rangle = \sum_{j,\sigma} B_{j,\sigma} |j, \sigma\rangle, \quad (29)$$

where $j = (L, R) = (0, 1)$. The joint probability of outputs x, y and successful post-selection is

$$\begin{aligned} \text{Prob}\{\Phi, x, y|\Psi, \rho_X, \rho_Y\} = & |C_0|^2 \Pi_X(x-1) \Pi_Y(y) \\ & + \Pi_X(X) \sum_{\sigma} |C_{1,\sigma}|^2 \Pi_Y(Y-\sigma) \\ & + \sum_{\sigma} [C_0 C_{1,\sigma}^* \rho_X(X-1, X) \rho_Y(Y, Y-\sigma) + c.c.] \\ & + \Pi_X(X) \sum_{\sigma} [C_{1,\sigma} C_{1,-\sigma}^* \rho_Y(Y-\sigma, Y+\sigma)] \Big\}. \quad (30) \end{aligned}$$

with $C_{j,\sigma} = B_{j,\sigma}^* A_{j,\sigma}$, $C_j = \sum_{\sigma} C_{j,\sigma}$. It is convenient to introduce the shorthand notation

$$M = |C_0|^2/2, \quad (31)$$

$$N_{\sigma} = |C_{1,\sigma}|^2, \quad (32)$$

$$N = \frac{N_+ + N_-}{2}, \quad (33)$$

$$Q = 2\text{Re} [C_{1,+} C_{1,-}^*], \quad (34)$$

$$R_{\sigma} = 2\text{Re} [C_0 C_{1,\sigma}^*] \quad (35)$$

$$R = \frac{R_+ + R_-}{2}. \quad (36)$$

As before, we consider the meters to be in a pure Gaussian state. Then the probability of post-selection is

$$\begin{aligned} \text{Prob}\{\Phi\} = & 2M + 2N + zQ + 2(wz)^{1/4}R \\ = & |\langle\Phi|\Psi\rangle|^2 + (z-1)Q + 2[(wz)^{1/4} - 1]R. \quad (37) \end{aligned}$$

The conditional probability is

$$\begin{aligned} Prob\{x, y|\Phi\} = \frac{\gamma\varepsilon}{2\pi Prob\{\Phi\}} & \left\{ 2M e^{-[\gamma^2(x-1)^2 + \varepsilon^2 y^2]/2} + \sum_{\sigma} N_{\sigma} e^{-[\gamma^2 x^2 + \varepsilon^2 (y-\sigma)^2]/2} \right. \\ & \left. + zQ e^{-\gamma^2 x^2/2 - \varepsilon^2 y^2/2} + (wz)^{1/4} e^{-\gamma^2 (x-1/2)^2/2} \sum_{\sigma} R_{\sigma} e^{\varepsilon^2 (y-\sigma/2)^2/2} \right\}, \end{aligned} \quad (38)$$

so that the characteristic function is

$$Z(\chi, \eta) = \frac{Z_0(\chi, \eta)}{Prob\{\Phi\}} \left\{ 2M e^{i\chi} + \sum_{\sigma} N_{\sigma} e^{i\sigma\eta} + zQ + (wz)^{1/4} \sum_{\sigma} R_{\sigma} e^{i(\chi + \sigma\eta)/2} \right\}. \quad (39)$$

The relevant averages are then

$$\langle x \rangle = \frac{2M + (wz)^{1/4} R}{Prob\{\Phi\}}, \quad (40)$$

$$\langle y^2 \rangle = \frac{1}{\varepsilon^2} + \frac{2N + (wz)^{1/4} R/2}{Prob\{\Phi\}}, \quad (41)$$

$$\langle xy^2 \rangle = \frac{\langle x \rangle}{\varepsilon^2} + \frac{(wz)^{1/4} R/4}{Prob\{\Phi\}}. \quad (42)$$

Thus, after eliminating $(wz)^{1/4} R$ and introducing $\mu = M/Prob\{\Phi\}, \nu = N/Prob\{\Phi\}$, the covariance is

$$\text{cov}(xy^2) = -\frac{1}{2} \langle x \rangle^2 + \left(\mu - 2\nu + \frac{1}{4} \right) \langle x \rangle - \frac{\mu}{2}. \quad (43)$$

This quadratic form may take positive values, provided that

$$\Delta^2 = \left(\mu - 2\nu + \frac{1}{4} \right)^2 - \mu > 0. \quad (44)$$

Furthermore, we want to avoid that $\langle x \rangle$ take negative values³, otherwise our criterion for a positive covariance as the footstep of the quantum Cheshire cat would be inapplicable, and we should consider, for instance, $\text{cov}(x^2 y^2)$. Thus we must have $\mu - 2\nu + \frac{1}{4} > 0$. We have already two conditions restricting the allowed coefficients, and hence the states $|\Psi\rangle$ and $|\Phi\rangle$. It is also clear that if w, z are close to zero, the correlation is necessarily negative (by definition $M \geq 0, N \geq 0$). These considerations have limited our hunting ground to a reasonable extent, and we can now find appropriate preparation and post-selection exhibiting a positive covariance.

³This is yet another surprising feature of postselected weak measurement. While the widely spread initial probability of the meter is shifted by +1 to the right when a photon is counted, the postselected average can be negative. Should we say that in this case we are witnessing a “negative presence” of the photon?

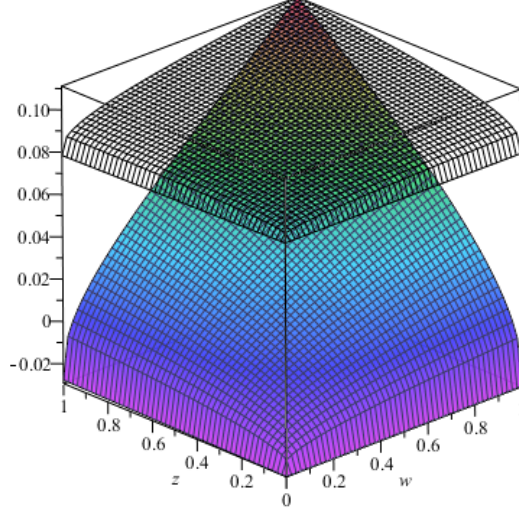


Figure 3: The covariance as a function of $w = \exp(-\gamma^2/2)$ and $z = \exp(-\varepsilon^2/2)$. The upper surface is the average $\langle x \rangle$ in arbitrary units.

As an example, consider

$$|\Psi\rangle = \sqrt{\frac{4}{5}}|L, +\rangle + \sqrt{\frac{1}{10}}|R, +\rangle + \sqrt{\frac{1}{10}}|R, -\rangle, \quad (45)$$

$$|\Phi\rangle = \sqrt{\frac{4}{5}}|L, +\rangle + \sqrt{\frac{1}{10}}|R, +\rangle - \sqrt{\frac{1}{10}}|R, -\rangle. \quad (46)$$

In Fig. 3, the covariance is plotted as a function of the interaction strengths w, z . It takes positive values, while $\langle x \rangle$ stays positive as well. Furthermore, the positive values do not require a very weak coupling, an intermediate one is sufficient. This concludes our hunt.

5 Conclusions

We have caught the quantum Cheshire cat, i.e., we have demonstrated that with a suitable preparation and post-selection of a photon, the joint measurements of its position in one region, with output x , and of its polarization in a separate region, with output y , shows a positive covariance $\text{cov}(xy^2)$. This is but one further manifestation of interference effects, that are preserved in the weak (or better, not so strong) coupling regime.

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